University of California, Berkeley
Physics H7A Fall 1998 (*Strovink*)

PROBLEM SET 9

1.

a. Expand in Taylor series: $f(x) = \ln{(1-x)}$; $f(x) = 1/(1+x)$.

b. Given two functions $s(x)$ and $c(x)$ such that $ds/dx = c$ and $dc/dx = s$, prove that $s(x) + c(x) = [s(0) + c(0)]e^x$.

2.

a. French problem 1-4(b).

b. French problem 1-9.

c. Prove DeMoivre's theorem,

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

3. French problem 3-15.

4. At $t = 0$ a bullet of mass m and velocity v_0 strikes a motionless block of mass M which is connected to a wall by a spring of constant k. The block moves with coordinate $x(t)$ (along the direction of the bullet) on a frictionless table next to the wall. The bullet embeds itself in the block. If $x(t) = \Re[\mathcal{A}\exp{(i(\omega t + \phi))}]$, with $\mathcal{A}$ real, evaluate ω, $\mathcal{A}$, and ϕ.

5. French problem 4-5.

6. French problem 4-8.

7. A piano has middle C = 256 Hz, and C-above-middle-C = 512 Hz. The white keys of its middle octave consist of middle C; D (1 step above middle C); E (2 steps); F (2.5); G (3.5); A (4.5); B (5.5); and C-above-middle-C (6 steps). Each step causes the frequency to be multiplied by a fixed factor.

a. Find the frequencies of D, E, F, G, A, and B.

b. If G were tuned to a "perfect fifth", its second harmonic ($2\times$ the fundamental frequency) would be the same as middle C's third harmonic. Find the *beat frequency* between the second harmonic of G and the third harmonic of middle C. (Pianos are tuned by listening for such beats.)

Stringed instruments are tuned (for compositions in the key of C) so that this beat frequency is zero, producing a smoother tone. When a composition in a different key is played, the stringed instrument can be retuned for that key, which would be impossible for the piano.

8. For an underdamped undriven harmonic oscillator with $\omega_0/\gamma \equiv Q = 100$, find the number of oscillations required to reduce the amplitude of oscillation by the factor $e^\pi \approx 23.1$.